

The Influence of Quintessence on the Separation of CMB Peaks

Michael Doran, Matthew Lilley, Jan Schwindt and Christof Wetterich
*Institut für Theoretische Physik der Universität Heidelberg
 Philosophenweg 16, D-69120 Heidelberg, Germany*

A hypothetical dark energy component may have an equation of state that is different from a cosmological constant and possibly even changing in time. The spacing of the cosmic microwave background peaks is sensitive to the ratio of the horizon sizes today and at last scattering and therefore to the details of the cosmological evolution. Together with independent measurements of today's cosmological parameters, this restricts quintessence models in the epoch before and during decoupling. For fixed Ω_0^b , Ω_0^{cdm} and h , we give an analytic estimate of the spacing that depends on Ω_{ls}^ϕ , Ω_0^ϕ and an effective equation of state of the dark energy component.

1 Introduction

Quintessence^{1,2} is a form of homogeneous energy associated with a scalar field having a time varying equation of state. If quintessence played a role in the epoch of last scattering, it may have left imprints in the Cosmic Microwave Background (CMB) fluctuations. The position of the m -th peak in the angular momentum spectrum of the CMB can be parameterized³ by

$$l_m = \Delta l(m - \phi_m),$$

where Δl is the average spacing between the peaks and the shift ϕ_m is a small quantity with absolute value less than 0.4, typically. For a flat universe and adiabatic initial conditions, Δl is given by the simple formula^{4,5}

$$\Delta l = \pi \frac{\tau_0 - \tau_{\text{ls}}}{s} = \pi \frac{\tau_0 - \tau_{\text{ls}}}{\bar{c}_s \tau_{\text{ls}}}. \quad (1)$$

Here τ_0 and τ_{ls} are the conformal time today and at last scattering (which are equal to the particle horizons) and $\tau = \int dt a^{-1}(t)$, with cosmological scale factor a . The sound horizon at last scattering s is related to τ_{ls} by $s = \bar{c}_s \tau_{\text{ls}}$, where the average sound speed before last scattering is approximately $\sqrt{1/3}$.

In this paper we will show that the spacing of the CMB peaks can be sensitive to quintessence.

2 Analytic estimate for Δl

We present here a quantitative discussion of the mechanisms which determine the spreading of the peaks. A simple analytic formula permits us to relate Δl directly to three characteristic quantities for the history of quintessence, namely the fraction of dark energy today, Ω_0^ϕ ,

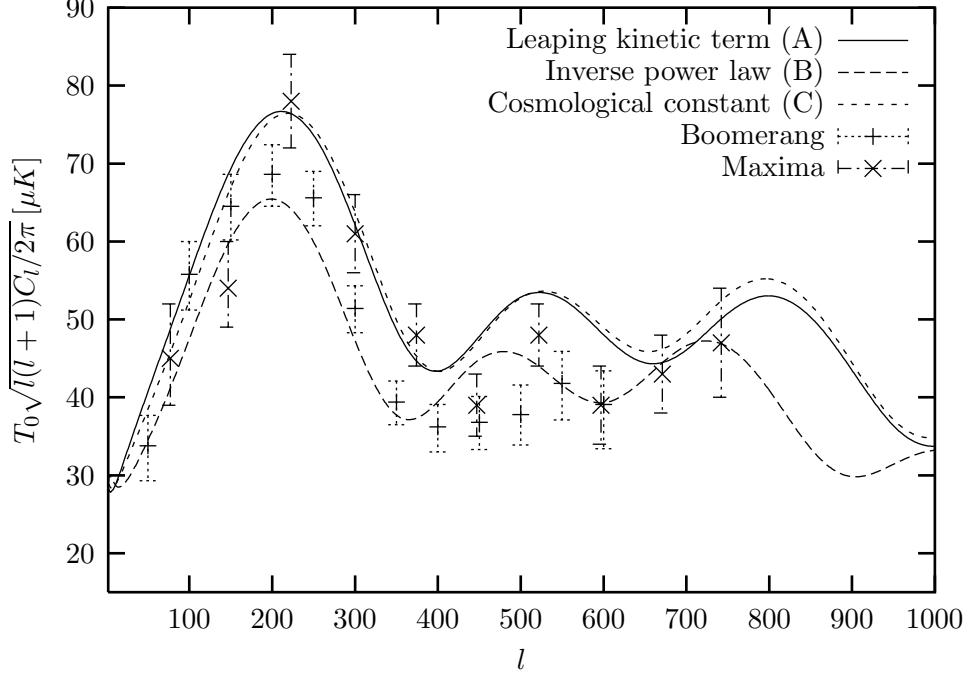


Figure 1: The CMB Spectrum for Λ -CDM (model C), leaping kinetic term (model A)⁶ and inverse power law (model B) quintessence universes with $\Omega_0^\phi=0.6$. The data points from the Boomerang⁷ and Maxima⁸ experiments are shown for reference.

the averaged ratio between dark pressure and dark energy, $\bar{w}_0 = \langle p^\phi / \rho^\phi \rangle_0$, and the averaged quintessence fraction before last scattering, $\bar{\Omega}_{ls}^\phi$ (for details of the averaging see Appendix). We compare our estimate with an explicit numerical solution of the relevant cosmological equations using CMB-FAST⁹. For a given model of quintessence the computation of the relevant parameters Ω_0^ϕ , \bar{w}_0 and $\bar{\Omega}_{ls}^\phi$ requires the solution of the background equations. Our main conclusion is that future high-precision measurements of the location of the CMB-peaks can discriminate between different models of dark energy if some of the cosmological parameters are fixed by independent observations (see fig. 1). It should be noted here that a likelihood analysis of the kind performed in¹⁰, where w is assumed to be constant throughout the history of the Universe, would not be able to extract this information as it does not allow $\bar{\Omega}_{ls}^\phi$ to vary. We point out that for time-varying w there is no direct connection between the parameters \bar{w}_0 and $\bar{\Omega}_{ls}^\phi$, i.e. a substantial $\bar{\Omega}_{ls}^\phi$ (say 0.1) can coexist with rather large negative \bar{w}_0 . We perform therefore a three parameter analysis of quintessence models and our work goes beyond the investigation for constant w in¹¹.

The average spacing is calculated¹² by integrating the Friedmann equation, assuming a constant fraction $\bar{\Omega}_{ls}^\phi$ of quintessence energy before last scattering and the effective equation of state \bar{w}_0 for later times:

$$\Delta l = \pi \bar{c}_s^{-1} \left[\frac{F(\Omega_0^\phi, \bar{w}_0)}{\sqrt{1 - \bar{\Omega}_{ls}^\phi}} \left\{ \sqrt{a_{ls} + \frac{\Omega_0^r}{1 - \Omega_0^\phi}} - \sqrt{\frac{\Omega_0^r}{1 - \Omega_0^\phi}} \right\}^{-1} - 1 \right], \quad (2)$$

with

$$F(\Omega_0^\phi, \bar{w}_0) = \frac{1}{2} \int_0^1 da \left(a + \frac{\Omega_0^\phi}{1 - \Omega_0^\phi} a^{(1-3\bar{w}_0)} + \frac{\Omega_0^r(1-a)}{1 - \Omega_0^\phi} \right)^{-1/2}, \quad (3)$$

and today's radiation component $\Omega_0^r h^2 = 4.2 \times 10^{-5}$, $a_{ls}^{-1} \approx 1100$ and $\bar{c}_s = 0.52$.

An alternative to the spacing between the peaks is the ratio of any two peak (or indeed trough) locations. After last scattering the CMB anisotropies simply scale ^a according to the geometry of the Universe – taking the ratio of two peak locations factors out this scaling and leaves a quantity which is sensitive only to pre-last-scattering physics. As can be seen in Table 1, (spatially-flat) models with negligible $\bar{\Omega}_{ls}^\phi$ all have $l_2/l_1 \approx 2.41$ for the parameters given in Table 1. The dependence of this ratio on the other cosmological parameters can be computed numerically ¹³, and thus a deviation from the predicted value could be a hint of time-varying quintessence.

$\bar{\Omega}_{ls}^\phi$	\bar{w}_0	l_1	l_2	l_2/l_1	$\Delta l^{\text{estim.}}$	$\Delta l^{\text{num.}}$	σ_8
Leaping kinetic term (A), $\Omega_0^\phi = 0.6$							
8.4×10^{-3}	-0.76	215	518	2.41	292	291	0.86
0.03	-0.69	214	520	2.43	294	293	0.78
0.13	-0.45	211	523	2.48	299	300	0.47
0.22	-0.32	207	524	2.53	302	307	0.29
Inverse power law potential (B), $\Omega_0^\phi = 0.6$							
8.4×10^{-8}	-0.37	199	480	2.41	271	269	0.61
9.9×10^{-2}	-0.13	178	443	2.49	252	252	0.18
0.22	-8.1×10^{-2}	172	444	2.58	257	257	0.09
Pure exponential potential, $\Omega_0^\phi = 0.6$							
0.70	7×10^{-3}	190	573	3.02	368	377	0.01
Pure exponential potential, $\Omega_0^\phi = 0.2$							
0.22	4.7×10^{-3}	194	490	2.53	282	281	0.38
Cosmological constant (C), $\Omega_0^\phi = 0.6$							
0	-1	219	527	2.41	296	295	0.97
Cold Dark Matter - no dark energy, $\Omega_0^\phi = 0$							
0	-	205	496	2.42	269	268	1.49

Table 1: Location of the first two CMB peaks l_1 , l_2 for several models of dark energy. We also show the analytic (from Equation (2) and numerical (from CMB-FAST) average spacing of the peaks, the ratio l_2/l_1 of the peak locations and σ_8 , the normalisation of the power spectrum on scales of $8h^{-1}\text{Mpc}$.

3 Conclusion

Structure formation restricts Ω^ϕ to be less than about 0.2 until very recently (see table 1). Big bang nucleosynthesis gives about the same bound. When MAP ¹⁴ has captured the CMB spectrum up to the third peak, it should - together with other means such as Supernovae data, clustering and lensing - be possible to narrow the space of cosmic parameters such that bounds on quintessence in a range of redshift $z = 10^5..10^3$ and on \bar{w}_0 can be derived. The phase shifts ϕ_m and hence the peak ratios will single out the recombination physics of quintessence, whereas Δl as a combination of both recombination and late time cosmology will help in determining \bar{w}_0 .

^aExcept for the ISW effect, which is slowly varying with l and rather small.

Appendix

The effective equation of state is defined as Ω^ϕ weighted τ average:

$$\overline{w}_0 = \int_0^{\tau_0} \Omega^\phi(\tau) w(\tau) d\tau \times \left(\int_0^{\tau_0} \Omega^\phi(\tau) d\tau \right)^{-1}. \quad (4)$$

Similarly, $\overline{\Omega}_{ls}^\phi \equiv \tau_{ls}^{-1} \int_0^{\tau_{ls}} \Omega^\phi(\tau) d\tau$ is just the τ average.

Table 2: Symbols, their meanings and numerical values used.

Symbol	Meaning	Value
$a(\tau)$	scale factor, normalised to unity today	
a_{ls}	scale factor at last scattering	1100^{-1}
h_0	Hubble parameter today $H_0 = 100 h_0$ km s $^{-1}$ Mpc $^{-1}$	0.65
Ω_0^r	relativistic Ω today	9.89×10^{-5}
Ω_0^b	baryon Ω today	0.05
\bar{c}_s	τ -averaged sound speed until last scattering	0.52
n	spectral index of initial perturbations	1

References

1. C. Wetterich, Nucl. Phys. **B302**, 668 (1988);
2. P.J.E. Peebles and B. Ratra, Astrophys. J. Lett. **325**, L17 (1988); B. Ratra and P.J.E. Peebles, Phys. Rev. **D37**, 3404 (1988)
3. W. Hu, M. Fukugita, M. Zaldarriaga and M. Tegmark, [astro-ph/0006436](#), Astrophys. J. in press.
4. W. Hu and N. Sugiyama, Astrophys. J. **444**, 489 (1995).
5. W. Hu, N. Sugiyama and J. Silk, Nature **386**, 37 (1997).
6. A. Hebecker and C. Wetterich, [hep-ph/0008205](#), to appear in Phys. Lett. **B**.
7. P. de Bernardis *et al.*, Nature **404**, 955 (2000).
8. S. Hanany *et al.*, [astro-ph/0005123](#).
9. U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).
10. J.R. Bond *et al.*, [astro-ph/0011379](#).
11. G. Huey, L. Wang, R. Dave, R.R. Caldwell and P.J. Steinhardt, Phys. Rev. **D59**, 063005 (1999).
12. M. Doran, M.J. Lilley, J. Schwindt and C. Wetterich, [astro-ph/0012139](#).
13. M. Doran and M. Lilley, [astro-ph/0104486](#)
14. <http://map.gsfc.nasa.gov>